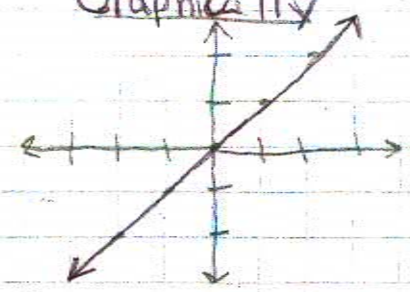
Patrick | Emily

#1

I. Graphs

- there are 3 ways to look at a graph.

Graphically



Table

x	y
-2	-2
-1	-1
0	0
1	1
2	2

Algebraically

$y = x$

Key elements

- There are several Key elements to look for

A.) Intercepts

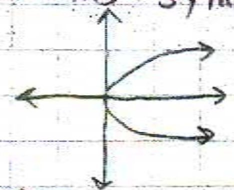
x-intercepts (set $x = 0$ and solve for y)
y-intercepts (set $y = 0$ and solve for x)

ex/
$x - y = 3$ x-intercept at $y = -3$
y-intercept at $x = 3$

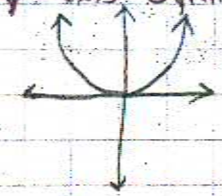
B) Symmetry - 3 types

x-axis symmetry (plug in y & $-y$, symm. occurs if equations are same)
y-axis symmetry (plug in x & $-x$, symm. occurs if equations are same)
origin symmetry (plug in (x,y) & $(-x,-y)$, " ")

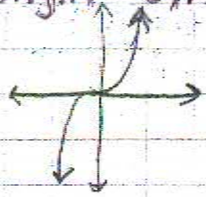
x-axis symm



y-axis symm



origin symm



C.) Intersections

i. substitution (solve for 1 variable and insert it into the other equation)

$y = x$ & $y = x^3$ $\rightarrow$ $x = x^3$
$0 = x(x-x)(x+x)$
$x = 0, 1, -1$ $\rightarrow$ $(0,0), (1,1), (-1,-1)$

ii Elimination (eliminate variable by adding equations)

$$\begin{array}{r} x + y = 2 \\ 2x - y = 1 \\ \hline 3x = 3 \\ x = 1 \end{array}$$

$(1,1)$

D.) Slope

P2 $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$

- If equations are parralel they have same slope
- if equations are perpendicular they have negative recip riedl slopes

$\left\{ \right.$ note: rate of change is slope except with different units from the x & y axis $\left. \right\}$

E.) Domain & Range

P3 Domain - set of all inputs
Range - set of all outputs

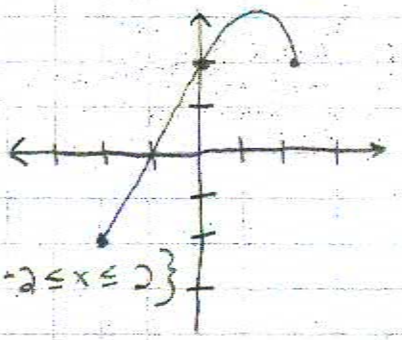
i Domain Restrictions
1. Zero's in denominator
2. Negative numbers under even roots
3. unidentified Regions

ii Range Restrictions - Look at the Graph

iii Graph Checklist
- ✓ assymptotes
- ✓ Turns
- ✓ Undefined Regions
- ✓ Piecewise Functions

ex/

$f(x) = x + \sqrt{4-x^2}$



Domain: $\{x \mid -2 \leq x \leq 2\}$

II Limits of Functions

A. Deffinition: what happens to x as you get close to y

$f(0) = \frac{\sin 0}{0}$ = undefined = bad

1.2 but you can find out what y is approaching

$\lim_{x \to 0} \frac{\sin x}{x} = 1$ the way you say it is the limit of $\frac{\sin x}{x}$ as x approaches 0 is 1

But how do you solve it?

B. Different ways to solve Limits

1. Look at a table

$\frac{\sin x}{x}$

x	-.1	-.01	-.001	0	.001	.01	.1
y	.998	.999	.9999998	DNE	.9999998	.999	.99

the $\lim_{x \to 0} \frac{\sin x}{x} = 1$

2. Plug and chug (only works when your not deviding by 0)

1.3 a) there are lots of properties that could help you but basically the main idea is just put the number x is approaching in your calculator/head and solve.

b) $\lim_{x \to 2} (4x^2 + 3) = 4(2)^2 + 3 = \boxed{19}$

3. 4 strategies to dividing by zero

a) Factor $\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = 0/0$

$\lim_{x \to 0} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}} = \boxed{3}$

b) Rationalize $\lim_{x \to 0} \frac{x}{\sqrt{x+1} - 1} = \frac{0}{0}$

$\lim_{x \to 0} \frac{x}{\sqrt{x+1} - 1} \cdot \frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1} = \frac{x(\sqrt{x+1} + 1)}{x + 1 - 1} = \sqrt{x+1} + 1$

$= \boxed{2}$

c) multiply by the LCD $\lim_{x\to 4} \frac{\frac{x}{x+1} - 4/5}{x-4}$

$$\lim_{x\to 4} \frac{\frac{x}{x+1} - 4/5}{\frac{x-4}{1}} \cdot \frac{5(x+1)}{5(x+1)} = \frac{5x-4(x+1)}{5(x+1)(x-4)} = \boxed{\frac{1}{25}}$$

d) Transform to known Limits

1. Don't forget (You must know these!!)

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \quad \& \quad \lim_{x\to 0} \frac{1-\cos x}{1} = 0$$

2 $$\lim_{x\to 0} \frac{\tan x}{x} = \frac{\frac{\sin x}{\cos x}}{\frac{x}{1}} \cdot \frac{1/x}{1/x} = \frac{\sin x}{x\cos x} = \frac{\sin x}{x} \cdot \frac{1}{\cos x}$$

$$= 1 \cdot 1 = \boxed{1}$$

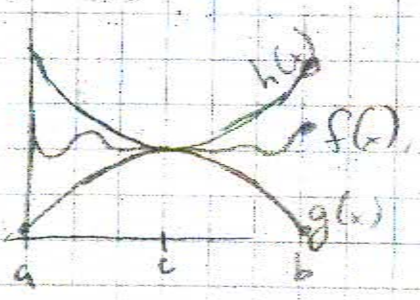
C. Squeeze Theorem

1. If $h(x) \le f(x) \le g(x)$ $\forall x$ in (a,b) containing c except possibly at c itself

and if $\lim_{x\to c} h(x) = L = \lim_{x\to c} g(x)$

then $\lim_{x\to c} f(x) = L$

2. Basically that means if $f(x)$ is between $h(x)$ and $g(x)$ and at a given point (c) the limit of both $h(x)$ and $g(x)$ are the same than $f(x)$ is squeezed between them and is also the same



here is a graph that shows it

III Asymptotes and unbounded behavior

A. Graphical Behavior of Asymptotes

<u>Vertical asymptotes</u> - are found where a graph is undefined

1.5 ex/ $y = \frac{1}{x}$ vertical asymptote at $x=0$

<u>Horizontal asymptotes</u> - are found by comparing the degrees of numerator and denominator

- if degree of top is larger then bottom there is no Horizontal asymptote
- if degree of top is smaller then bottom Horizontal asymptote is at $y=0$
- if degree of top is equal to bottom Horizontal asymptote is at the coefficient of top over bottom

ex/ $f(x) = x^3$ no asymptotes

$f(x) = \frac{1}{x}$ V.A. at $x=0$ H.A. at $y=0$

$f(x) = \frac{x^2+4}{2x^2-4}$ V.A. at $x = \pm\sqrt{2}$ H.A. at $y = \frac{1}{2}$

B.) Asymptote involving limits to infinity

if $\lim_{x\to\infty} f(x) = L$ then $Y = L$ is a H.A.

if $\lim_{x\to c} f(x) \Rightarrow \infty$ then $x = C$ is a V.A.

C) Comparing Exponential, Polynomial, and logarithic growth

- Logarithmic begin with greatest growth but eventually levels off
- Polynomial begins after logarithmic with a higher rate of growth then exponential
- eventually Exponential has the greatest growth rate but is the last to achieve it.

IV Continuity as a property of functions

A Def: Continuity = an unbroken graph with no holes, or gaps and usually no vertical asymptotes.

1.4

B. A function is continuous at a point if...

1) $f(c)$ is defined

2) $\lim_{x \to c} f(x)$ exists

3) $\lim_{x \to c} f(x) = f(c)$

C. An entire function is continuous if...

1) it is continuous at every point...(duh)

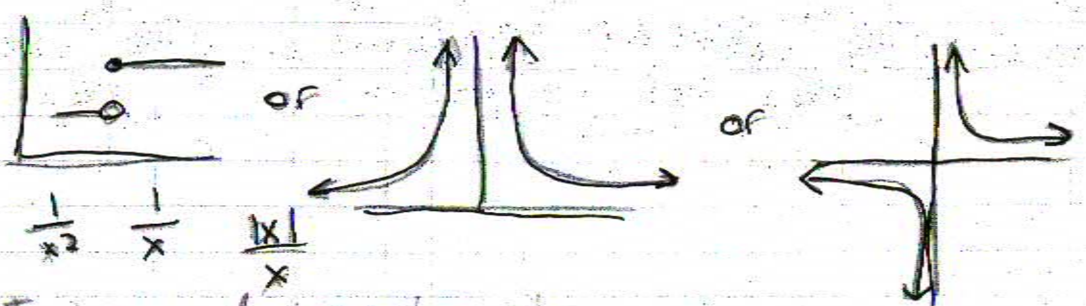
2) $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)$

D One sided Limits

1) $\lim_{x \to c^+} f(x) \Rightarrow$ lim from the right side of the function
or positive side

2) $\lim_{x \to c^-} f(x) \Rightarrow$ limit from the left side of the function
or negative side

3) used most ~~often~~ often in functions like this



$\frac{1}{x^2}$ $\frac{1}{x}$ $\frac{|x|}{x}$

E) Intermediate Value Theorem

1. If $f(x)$ is continuous on $[a,b]$ and k is any number between $f(a)$ and $f(b)$ then there exists c contained in (a,b) such that $f(c) = k$

2. Basically this means that if $f(x)$ is continuous on a closed interval then every height and width (x & y) between those 2 points is touched

3. ex $f(x) = x^3 - 3x^2 + 1$ prove using IVT the function crosses the x-axis between $[0, 1]$

$f(0) = 1$

$f(1) = -1$

since $f(x)$ is continuous on $[0, 1]$ and $f(0)$ is positive and $f(1)$ is negative there is c contained in $(0, 1)$ whee $f(c) = 0$

F. Extreme Value Theorem

1. If f is continuous from a to b then f has a minimum and a maximum

2. In other words duh